Jet-Damping Effects: Theory and Experiment

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A rocket pitching or yawing about a transverse axis (Fig. 1) has a loss in angular momentum with respect to its center of mass. This loss is a result of the transverse velocity component of the escaping gases and can be written as $l_e^2(dm/dt)\omega_x$; l_e is defined in Fig. 1, ω_x is angular velocity about a transverse axis, and dm/dt is the mass rate of flow of escaping gases. This loss in angular momentum we define as jet damping.

In studying the effects of jet damping, Rosser, Newton, and Gross[1] included the effects of the change in moment of inertia of the vehicle in combination with the preceding angular momentum loss and defined the combination as jet damping. Recently, Rott and Pottsepp[2] analyzed the problem from the fluid dynamic viewpoint, including the Coriolis effect in their equations of motion. They integrated the pressure forces and moments to obtain a resultant jet-damping moment that consisted of the jet-damping term defined previously plus a second term that can be interpreted as a time rate of change in moment of inertia. Their analysis shows that jet damping is a result of Coriolis forces.

Thomson[3] examines the problem from the point of view of Rosser, Newton, and Gross. For the special case of a radially-burning solid rocket, with l_e and the radius of gyration k assumed to remain constant, he integrates his equation of motion to obtain

$$\omega = \omega_0(m/m_0)^{(l_e/k)^2-1} \tag{1}$$

This result shows that for $l_e/k > 1$, ω decreases, whereas for $l_e/k < 1$, ω increases. Thus, there is a possibility of instability in jet-dampened rockets. In a later section Thomson formulates the general governing equations for motion of spinning bodies with varying configuration and mass. He writes his moment equation in three separate forms, the first of which contains the Coriolis term but does not contain the $dm/dt\ \omega l_e^2$ or dI/dt terms. By addition and substraction of the jet damping and other necessary terms, he puts the governing moment equation in the form shown in Eq. (2) below. If one works with Thomson's general formulation in its first form and makes the same internal flow assumptions as Rott and Pottsepp, their results are obtained. In the present note the problem is formulated from Eq. (2) because it leads to results that are easily interpreted and that to the author's knowledge have not been presented elsewhere. The results obtained are applied to a particular experimental configuration that was designed to demonstrate the instability condition.[4]

Theoretical Development

Referring to Fig. 1, we consider a single exhaust, axially symmetric vehicle. We further take the nonspinning condition $\omega_z = 0$ so that we are interested in determining the change in ω_x or ω_y from some small initial value of either one of these quantities. Because the center of mass of a variable mass rocket will not in general be a body-fixed point, we choose to formulate the problem in terms of the inertial and body fixed coordinates shown in Fig. 1. The system is defined as a group of particles within a specified boundary with body coordinates x, y, z. Following Thomson, the basic moment and force equations for this system can be written

$$\begin{aligned}\mathbf{M}_0 = &-(d^2\mathbf{R}_0/dt^2) \times m\,\mathbf{r}_0 + d(\mathcal{I}\cdot\boldsymbol{\omega})/dt + \\ &\boldsymbol{\omega} \times \Sigma\mathbf{r}_i \times m_i[d\mathbf{r}_i/dt] + \Sigma\mathbf{r}_i \times m_i[d^2\mathbf{r}_i/dt^2] - \\ &\Sigma\,\mathbf{r}_i \times (dm_i/dt)\mathbf{U}_i - \Sigma\mathbf{r}_i \times (dm_i/dt)(\boldsymbol{\omega}\times\mathbf{r}_i)\end{aligned} \tag{2}$$

and

$$\begin{aligned}\mathbf{F} = m\{&(d^2\mathbf{R}_0/dt^2) + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}_0) + (d\boldsymbol{\omega}/dt)\times \\ &\mathbf{r}_0 + 2\boldsymbol{\omega}\times[d\mathbf{r}_0/dt] + [d^2\mathbf{r}_0/dt^2]\} - \Sigma(dm_i/dt)\mathbf{U}_i\end{aligned} \tag{3}$$

where

$\mathbf{M}_0$ = moment of external forces about body-fixed orgin
m = mass of vehicle at any instant
$\mathbf{r}_0$ = position vector of c.m. in body-fixed system
$\mathcal{I}$ = inertia diadic of system at any instant
$\boldsymbol{\omega}$ = angular velocity vector of system
$\mathbf{r}_i$ = position vector of ith particle of system
m_i = mass of ith particle of system
$\mathbf{U}_i$ = velocity of particles leaving system relative to system
dm_i/dt = mass rate of flow out of system
$\mathbf{F}$ = forces external to system

Equation (2) contains the Coriolis effects of the gases in both the second and the third terms (see Thomson[3] for complete derivation). We set the 3rd, 4th, and 5th terms of Eq. (2) equal to zero from the axial symmetry condition imposed on the problem. This condition forces $\mathbf{r}_i$, $d\mathbf{r}_i/dt$, $d^2\mathbf{r}_i/dt^2$, and $\mathbf{U}_i$ to be parallel vectors. (There may be practical cases where this wouldn't be true, i.e., canted engines or swirling nozzles.) Since we are interested in small changes in $\boldsymbol{\omega}$ under zero external forces and moments we set $\mathbf{M}_0$ and $\mathbf{F}$ equal to zero.

Equation (3) can be solved for $\mathbf{R}_0$ and the result substituted into Eq. (2); this leads to

$$\begin{aligned}0 = -\{&-\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}_0) - \boldsymbol{\omega}\times\mathbf{r}_0 - 2\boldsymbol{\omega}\times \\ &[d\mathbf{r}_0/dt] - [d^2\mathbf{r}_0/dt^2] - \Sigma(dm_i/dt)\mathbf{U}_i/m\}\times m\,\mathbf{r}_0 + \\ &[d(\mathcal{I}\cdot\boldsymbol{\omega})/dt] - \Sigma\mathbf{r}_i\times(dm_i/dt)(\boldsymbol{\omega}\times\mathbf{r}_i)\end{aligned} \tag{4}$$

Then because of axial symmetry and no spinning we have

$$\mathbf{r}_0 = -r_0\,\mathbf{e}_z \qquad [d\mathbf{r}_0/dt] = -(dr_0/dt)\mathbf{e}_z$$
$$[d^2\mathbf{r}_0/dt^2] = (d^2r_0/dt^2)\mathbf{e}_z \qquad \boldsymbol{\omega} = \omega_x\mathbf{e}_x$$
$$\mathbf{U}_i = U_i\mathbf{e}_z \qquad \mathcal{I} = I_{xx}\mathbf{e}_x\mathbf{e}_x + I_{yy}\mathbf{e}_y\mathbf{e}_y + I_{zz}\mathbf{e}_z\mathbf{e}_z$$
$$\mathbf{r}_i = -l_e\mathbf{e}_z$$

where $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ represent unit vectors. Using these values, Eq. (4) can be rewritten as

$$[m\,r_0^2(d\omega_x/dt) + 2\,m\,r_0(dr_0/dt)\omega_x]\mathbf{e}_x = [I_x\cdot(d\omega_x/dt) + (dI_x/dt)\omega_x + l_e^2(dm/dt)\omega_x]\mathbf{e}_x$$

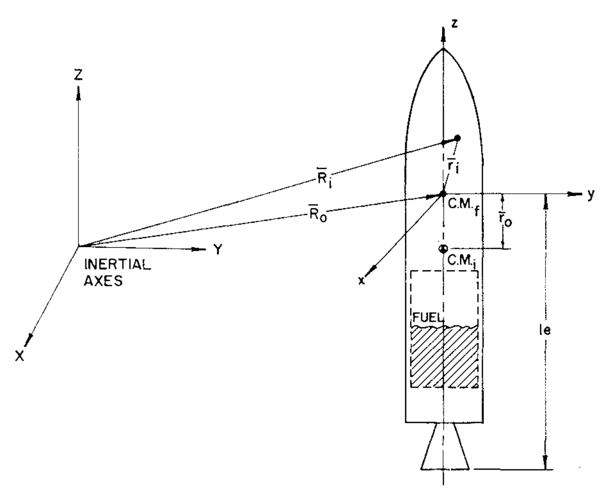

Fig. 1 Model for analysis of rocket with moving center of mass.

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or, since this is a scalar equation

$$[(I_x - m\,r_0^2)(d\omega_x/dt)/\omega_x] - 2\,m\,r_0(dr_0/dt) + (dI_x/dt) - l_e^2 dm/dt = 0 \quad (5)$$

where we now recognize the term $(I_x - mr_0^2)$ as the instantaneous moment of inertia with respect to an axis through the center of mass. Dividing through $(I_x - mr_0^2)$ gives

$$(d\omega_x/dt)/\omega_x - [2mr_0(dr_0/dt) - dI_x/dt + l_e^2 dm/dt]/(I_x - mr_0^2) = 0$$

where, if we now choose to take our body-fixed orgin always coinciding with the center of mass $r_0 = 0$, we get

$$(d\omega_x/dt)\omega_x + (dI_x/dt)/I_x - l_e^2(dm/dt)/I_x = 0 \quad (6)$$

In Eq. (6), l_e is now a variable, and I_x is the instantaneous moment of inertia about an x axis through the center of mass. Multiplying Eq. (6) through by dt, it may be integrated to give

$$\frac{\omega}{\omega_0} = \frac{I_0}{I}\exp\left(\int_{m_0}^{m}\frac{l_e^2}{I}\,dm\right) \quad (7)$$

where the x subscript has been dropped, and the subscript 0 refers to initial conditions. For the special case $l_e = 0$, we obtain the usual conservation-of-angular-momentum result,

$$\omega I = \omega_0 I_0 = \text{const} \quad (8)$$

This shows an unstable situation since the moment of inertia is decreasing. A second case of interest is the one solved by Thomson where l_e and K, the radius of gyration, are both assumed constant. Equation (7) can then be written

$$\frac{\omega}{\omega_0} = \frac{I}{I_0}\exp\left(l_e^2\int_{m_0}^{m}\frac{dm}{k^2 m}\right) = \frac{m_0k^2}{mk^2}\exp\left(\frac{l_e^2}{k^2}\int_{m_0}^{m}\frac{dm}{m}\right)$$

which integrates to

$$\omega/\omega_0 = m_0/m\,(m/m_0)^{(l_e^2/k^2)} = (m/m_0)^{(l_e^2/k^2 - 1)} \quad (9)$$

the result obtained by Thomson.

Thus we see that the behavior of a jet-dampened rocket under zero torque conditions is dependent upon the integral in Eq. (7) which must be solved for each individual case. It is noted that the jet-damping term $l_e^2(dm/dt)\omega$ is always stabilizing, and instability occurs only when the destabilizing effects of the loss in the moment of inertia exceeds the jet-damping term.

Experimental Apparatus and Results

To experimentally verify the preceding theoretical results, a model was constructed and tested. The model was specifically designed to demonstrate the instability predicted by the theory. Figure 2 is a schematic of the model. It consisted of two CO_2 fire-extinguisher bottles mounted in a steel framework. Both bottles fed their gas into a single pressure regulator, which in turn fed the nozzle. As shown in Fig. 2, the bottles are symmetric with the axis of rotation x, and lie along the y axis. The nozzle is in the negative z direction. The configuration rotated on a vertical shaft in

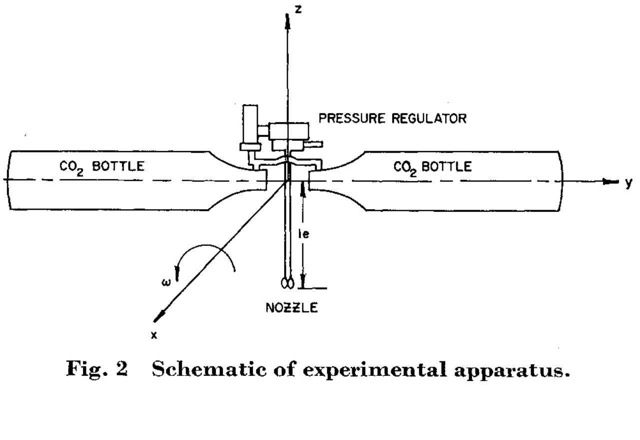

Fig. 2 Schematic of experimental apparatus.

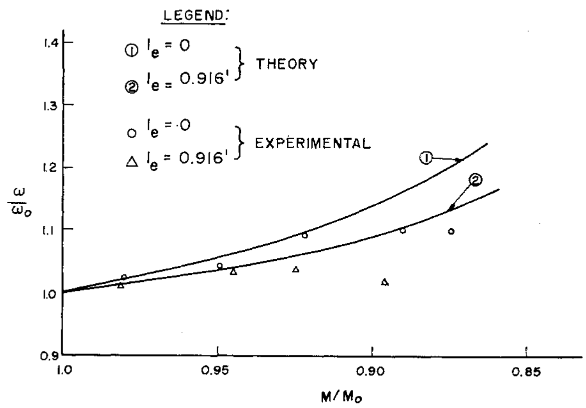

Fig. 3 Comparison of theoretical and experimental results.

a horizontal plane. For the case of constant angular momentum $l_e = 0$ the nozzle was turned and directed so that it discharged along the positive x axis, the axis of rotation. For both configurations, the center of mass of the system remained fixed on the axis of rotation, and the distance l_e was a constant. Because of the configuration, the moment of inertia I_x varied with the speed of rotation as well as with the mass flow. The effect of the rotation on the moment of inertia was accounted for by computing the parabolic free surface of the liquid CO_2 as a function of ω and remaining CO_2. The effect of rotation on the moment of inertia required that all experimental runs that were to be compared be made with the same initial ω.

Because friction could not be eliminated from the system, its effect was experimentally determined by the following two runs with no mass flow: 1) with the bottles empty and 2) with the bottles full. Then with the mass ratio m/m_0 a known function of time, an average ω/ω_0 due to friction was plotted vs m/m_0. This frictional effect was then subtracted from the experimental jet-damping curves to correct them to the torque-free condition. Experimental runs were made with $l_e = 0.916$ ft and $l_e = 0$, both with the same initial ω_0.

Figure 3 compares the theoretical and experimental results. The upper solid curve is for $l_e = 0$, angular momentum conserved, whereas the lower solid curve is for $l_e = 0.916$ ft. The difference between these two curves is the effect of jet damping. The corrected experimental data points follow the theoretical curves quite closely for $0.95 \leq m/m_0 \leq 1$ but deviate from the theoretical curve for $m/m_0 < 0.95$. Attempts to isolate the cause of the deviation were unsuccessful. Several factors were observed which could influence the behavior of the system for $m/m_0 < 0.95$. These factors were as follows: a) Considerable reduction in flow rate occurred with the loss in internal pressure, simultaneously resulting in unsteady flow conditions due to ejection of frozen CO_2 particles (generally $\frac{1}{3}$ the total mass loss occurred in the first 1 $\frac{1}{2}$ min, the next $\frac{1}{3}$ in the following 3 $\frac{1}{2}$ min, whereas the final $\frac{1}{3}$ required nine additional minutes); and b) Changes in bearing friction occured as a result of changes in temperature, thrust, and unequal bottle weights because of unequal flow rates between the bottles. The experimental results demonstrate the instability condition and provide experimental verification of Eq. (7) for the initial part of the flow.

References

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[2] Rott, N. and Lembit, P., "Simplified calculations of jet damping effect," AIAA J. **2**, 764–766 (1963).

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